International Journal of Thermophysics, Vol. 23, No. 3, May 2002 (© 2002)

Estimation of the Mean Thermal Conductivity of Anisotropic Materials

T. Ohmura,[1,2] M. Tsuboi,[1] and T. Tomimura[3]

Received November 19, 2001

An estimation method of the plane directional thermal conductivity of fibrous insulations using the cyclic heat method and the transient hot-wire method is proposed. By assuming that the thermal conductivity λ_h of anisotropic materials measured by the transient hot-wire method is equivalent to that of the isotropic materials which have the same bulk density ρ and specific heat c as the anisotropic materials, the thermal conductivity λ_h is shown to be equal to $\sqrt{\lambda_x \lambda_y}$, which is a geometrical mean of the thermal conductivities in the direction of the plane λ_x and the thickness λ_y of the anisotropic materials. For an alumina silica blanket ($\rho = 125\ \mathrm{kg \cdot m^{-3}}$), the thermal conductivities λ_h, λ_x, and λ_y were measured in the temperature range between -140 and 300°C using the transient hot-wire method for λ_h and the cyclic heat method for λ_x and λ_y. In the same way, the thermal conductivities λ_h, λ_x, and λ_y of a rock wool ($\rho = 121\ \mathrm{kg \cdot m^{-3}}$) insulation were also measured in the temperature range, 100 to 600°C. From a comparison of the measured results with the estimated values of λ_x, it is confirmed that the proposed method can estimate the measured values reasonably well.

KEY WORDS: alumina silicate, anisotropic material, cyclic heat method, rock wool, thermal conductivity, transient hot-wire method

1. INTRODUCTION

In general, thermal insulations are classified into the following two types of materials: isotropic materials with a uniform thermal conductivity and anisotropic materials with a nonuniform thermal conductivity. Examples of

[1] Research and Development Department, Hamamatsu Research Center, Nichias Corporation, 1-8-1 Shin-Miyakoda, Hamamatsu, Shizuoka 431-2103, Japan.
[2] To whom correspondence should be addressed. E-mail: oomura-t@nichias.co.jp
[3] Institute of Advanced Material Study, Kyushu University, 6-1 Kasuga-Koen, Kasuga, Fukuoka 816-8580, Japan.

0195-928X/02/0500-0843/0

isotropic materials are polyurethane foam, calcium silicate, etc., whereas fibrous insulations are examples of the anisotropic materials. Fibrous insulations are widely used in many industries, including the space, power, and building construction industries. For construction purposes, thermal insulations are used in various ways, namely, depending on the application, they are piled either perpendicular or parallel to heat flow. In the case of fibrous insulations, which are composed of laminated fleeces, the insulation performance is largely affected by the different applications mentioned above, since the thermal conductivities in the direction of the plane (parallel to the oriented fibers) and the thickness λ_x, λ_y are quite different from each other. However, because the performance of such fibrous insulations is generally evaluated with the thermal conductivity λ_y, the plane directional thermal conductivity λ_x is rarely measured except when the specific value is required. Accordingly, when it comes to measuring the thermal conductivity λ_x of fibrous insulations, lots of problems are encountered. For example, in preparation of test specimens, one needs to cut the insulations into strips and rotate them by 90°, but each strip may not have the same thickness because of cutting error. In addition, the strips easily peel off in thin sheets, leading to difficulty to obtain uniform thickness and the same bulk density as the original insulations, etc. In some cases, extreme deformation of the fibrous structure will be experienced, and furthermore, an appropriate method for joining each strip and reducing the clearance remaining at the spliced surfaces of the strips has to be developed.

To cope with those problems, an estimation method of the plane directional thermal conductivity λ_x of the fibrous insulations using the cyclic heat method and the transient hot-wire method is proposed in this study. In the present method, the thermal conductivity λ_x is estimated by using both the thermal conductivity λ_h measured by the transient hot-wire method, in which the directional effect of the thermal conductivity is represented, and the thermal conductivity λ_y measured by the cyclic heat method. Those two kinds of measurements by the cyclic heat method and the transient hot-wire method were performed with an integrated measuring system that has been developed by the authors [1]. Jackson et al. [2] have also proposed a similar estimation method. However, since they used only the transient hot-wire method, they could not measure the thermal conductivity in the direction of the thickness λ_y. Then, they obtained the plane directional thermal conductivity λ_x by carrying out hot-wire measurements with the wire placed perpendicular to the fiber planes. Furthermore, by measuring λ_h in the same manner, they estimated the thermal conductivity λ_y. The estimated results of the thermal conductivity λ_y agreed with the values measured by the guarded hot-plate technique. However, an elliptic integral of the first kind is included in their equation; thus, their method is more complicated and inconvenient than the method of this study.

2. ESTIMATION METHOD OF PLANE DIRCTIONAL THERMAL CONDUCTIVITY

Figure 1 shows the physical model and coordinate system for an anisotropic material. The x- and z-axes are contained in the plane, which is parallel to the oriented fibers, and the y-axis is perpendicular upward to the plane. A hot wire is placed along the z-axis. Here, $s = s(x, y, t)$ shows the rim of the temperature field. When the material is isotropic in the x–z plane, the unsteady state heat conduction equation for this system is given by

$$\frac{\partial T}{\partial t} = \kappa_x \frac{\partial^2 T}{\partial x^2} + \kappa_y \frac{\partial^2 T}{\partial y^2} \tag{1}$$

where T is the temperature and t is the time. κ_x and κ_y are the thermal diffusivities in the x- and y-directions and are defined by

$$\kappa_x = \frac{\lambda_x}{\rho c} \tag{2a}$$

$$\kappa_y = \frac{\lambda_y}{\rho c} \tag{2b}$$

Here, $\lambda_x, \lambda_y, \rho$, and c are the thermal conductivities in the x- and y-directions, the bulk density, and the specific heat, respectively.

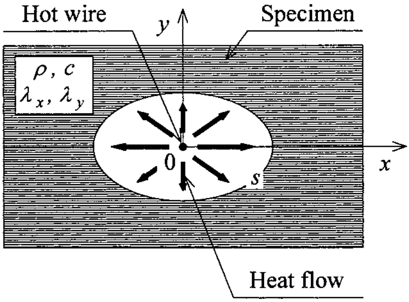

Fig. 1. Physical model and coordinate system of anisotropic material.

On the other hand, the thermal conductivity λ_h of an anisotropic material measured by the transient hot-wire method is considered to be equivalent to the thermal conductivity of an imaginary isotropic material which has the same ρ and c as those of the anisotropic material. The transient hot-wire method is a way of measuring thermal conductivity which assumes that heat generated on the hot wire conducts in a radial direction uniformly. If we consider the Cartesian coordinates, $\tilde{x}$ and $\tilde{z}$ in a plane, and $\tilde{y}$ extending perpendicularly upward, the unsteady-state heat-conduction equation for this system is given by

$$\frac{\partial T}{\partial t} = \kappa_h \left(\frac{\partial^2 T}{\partial \tilde{x}^2} + \frac{\partial^2 T}{\partial \tilde{y}^2} \right) \tag{3}$$

Here, the thermal diffusivity κ_h, in which the effect of an anisotropic characteristic is included, is defined by

$$\kappa_h = \frac{\lambda_h}{\rho c} \tag{4}$$

Equations (1) and (3) give identical results provided that the mean temperature responses of the hot wire are considered, and the equations can be transformed with each other by the following Eqs. (5a) and (5b).

$$\tilde{x} = \sqrt{\frac{\kappa_h}{\kappa_x}}\, x \tag{5a}$$

$$\tilde{y} = \sqrt{\frac{\kappa_h}{\kappa_y}}\, y \tag{5b}$$

Next, since the total quantity of heat, which is given out by the hot wire and is stored in the material during the time $0 \sim t$ in the $\tilde{x}$–$\tilde{y}$ system, is equal to the total quantity of heat in the x–y system, the following condition is required to be satisfied:

$$\int_s \rho c \{T(x, y, t) - T_\infty\}\, dx\, dy = \int_{\tilde{s}} \rho c \{T(\tilde{x}, \tilde{y}, t) - T_\infty\}\, d\tilde{x}\, d\tilde{y} \tag{6}$$

Here, T_∞ is the initial temperature at the time $t = 0$. By substituting Eqs. (5a) and (5b) into the right-hand side of Eq. (6), the following condition expressed by the x–y system is obtained,

$$\int_s \rho c \{T(x, y, t) - T_\infty\}\, dx\, dy = \frac{\kappa_h}{\sqrt{\kappa_x \kappa_y}} \int_s \rho c \{T(x, y, t) - T_\infty\}\, dx\, dy \tag{7}$$

From the above equation, the following relation is obtained among the thermal conductivities λ_h, λ_x, and λ_y by substituting Eqs. (2a), (2b), and (4) into Eq. (7).

$$\lambda_h = \sqrt{\lambda_x \lambda_y} \tag{8}$$

Here, Eq. (8) is also derived as follows. Since the area bounded by the edge of the temperature field at a given time t is the same in both the x–y and $\tilde{x}$–$\tilde{y}$ systems, the following condition is obtained.

$$\int_s dx\, dy = \int_{\tilde{s}} d\tilde{x}\, d\tilde{y} \tag{9}$$

By substituting Eqs. (5a) and (5b) into the right-hand side of Eq. (9), Eq. (8) is obtained.

On the other hand, Jackson et al. [2] derived a relation among λ_h, λ_x, and λ_y by assuming that the thermal conductivity λ_h is the average of the thermal conductivity $\lambda(\phi)$ over the cylindrical polar angle ϕ, where the hot wire represents the axis of the cylinder, and, in addition, by assuming that $\lambda(\phi)$ varies elliptically with ϕ, with λ_x and λ_y the semimajor and semiminor axes, respectively. Jackson et al. proposed the following equation:

$$\lambda_h = \frac{2\lambda_y}{\pi} \int_0^{\pi/2} (1 - m \sin^2 \phi)^{-1/2}\, d\phi \tag{10}$$

where

$$m = \frac{\lambda_x^2 - \lambda_y^2}{\lambda_x^2} \tag{11}$$

3. MEASUREMENTS

In order to check the validity of Eq. (8) on the basis of measured results, the thermal conductivity λ_h of an anisotropic material is measured by the transient hot-wire method, and λ_x, λ_y by the cyclic heat method over a temperature range from −140 to 300°C. For temperatures below 100°C, a measuring system [1] that employs the cyclic heat method, the transient hot-wire method, and the hot disk method [3] was used. Above 100°C, a previously developed apparatus that employs the cyclic heat method [4] was used after modifications and the addition of a hot wire.

Figure 2 shows a schematic of the measuring system, which can be used to perform three kinds of thermal conductivity measurements [1].

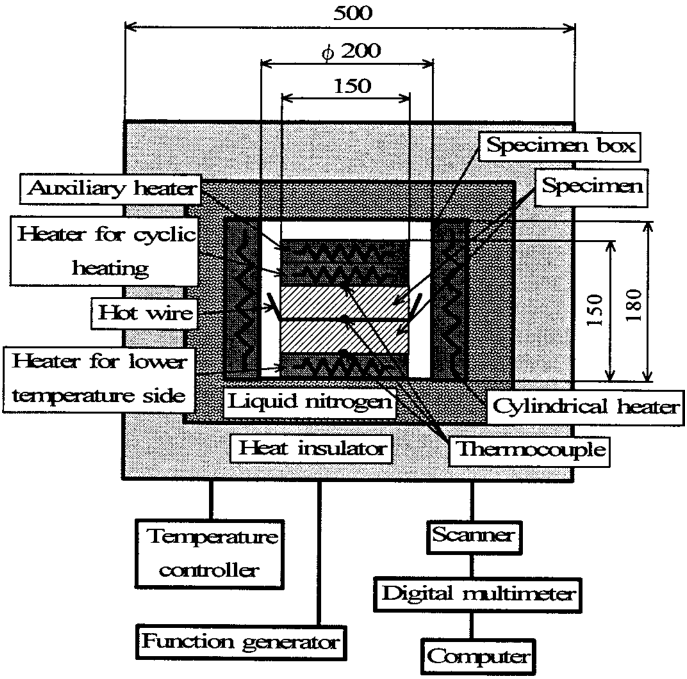

Fig. 2. Schematic of apparatus for thermal conductivity measurement of thermal insulations. All dimensions are given in mm.

The system is composed of a cold reservoir, the measurement cell, and a control system. The specimens and their surroundings are cooled by liquid nitrogen. The cold reservoir consists of the liquid nitrogen tank and the surrounding urethane foam insulation. The measurement cell is made of the aluminum specimen box and the cylindrical heater for forming an ambient temperature condition. In the specimen box, there are three kinds of heaters, that is, the heater for cyclic heating to generate an arbitrary temperature wave with a function generator, the auxiliary heater to transmit the temperature wave effectively to the specimen, and the heater for the lower temperature side. A pair of specimens of each rectangular prism with a base of 150×100 mm and a height of 20 mm is placed between the heaters for cyclic heating and the lower temperature side. The hot wire of 0.3 mm in diameter and 150 mm in length (a nichrome wire with an electrical resistance of 2.5 Ω) and a K-type thermocouple of 0.1 mm in diameter are inserted between the two specimens. The control system is composed of a temperature controller, a function generator, a digital multimeter, a power supply, and a personal computer. In the measurements by the cyclic heat method, a temperature wave with a period of 2 h and a voltage amplitude of 1 V (which corresponds to a temperature amplitude of 4°C) was generated. Meanwhile, in the case of the transient hot-wire method, an electric power was supplied step-wise to the nichrome wire with a voltage of 0.5 V for a duration of 10 minutes. The arrangement of the apparatus for the cyclic heat method used above 100°C is similar to that shown in Fig. 2, and the measurements were conducted by inserting the same hot wire between the two specimens. In the cyclic heat method, the specific heat of the specimen is required to evaluate the thermal conductivity. Thus, the specific heat was measured by the hot disk method below 100°C, and by a drop calorimeter [4, 5] above 100°C.

4. RESULTS AND DISCUSSION

First, the thermal conductivities of the isotropic materials of polyurethane foam and calcium silicate were measured using the cyclic heat and transient hot-wire methods to confirm that the measurements provided reliable results. Next, for the alumina silica blanket the thermal conductivities in the direction of the plane and the thickness, λ_x and λ_y, were measured by the cyclic heat method. Then, the thermal conductivity λ_h, which includes the effect of anisotropic behavior, was measured by applying the transient hot-wire method. Finally, the validity of the present estimation method was examined through comparisons between the measured results on λ_x and the results estimated from Eq. (8).

4.1. Thermal Conductivities of Isotropic Materials

Figure 3 shows the measured thermal conductivity λ of the polyurethane foam (bulk density, $\rho = 74$ and $119\,\mathrm{kg \cdot m^{-3}}$). Here, the closed circle and closed triangle symbols represent the results obtained by the cyclic heat method and by the transient hot-wire method, respectively. To facilitate comparisons, results from the transient hot-wire method are fitted with a quadratic equation using the least squares method, and the fit shown by the solid line. The broken lines indicate $\pm 10\%$ deviations from the solid line. It is seen from Fig. 3 that the measured values in the temperature range of -180 to 25°C can be fitted to within $\pm 10\%$ scatter. Here, the results obtained by the cyclic heat method deviate significantly from the solid line at around 0°C. The reason for this large deviation could be attributed to the phase change of water contained in the specimens [1].

The thermal conductivity λ of calcium silicate (bulk density, $\rho = 123$ $\mathrm{kg \cdot m^{-3}}$) is shown in Fig. 4. Here, the solid line represents a linear fit of the results from the transient hot-wire method; the meaning of each symbol

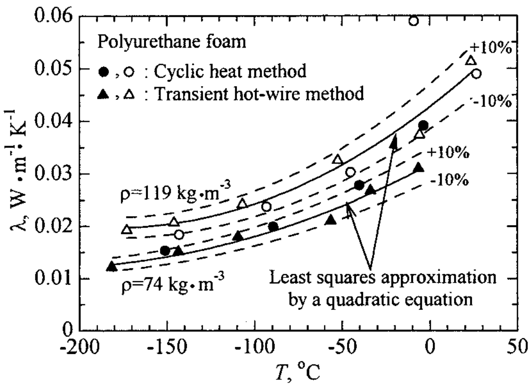

Fig. 3. Thermal conductivity λ of polyurethane foam with a bulk density $\rho = 74$ and 119 kg·m^{-3}. ●, ○ measured by the cyclic heat method; ▲, △ measured by the transient hot-wire method; least squares fit of the measured results; – – – ± 10% deviations from solid line.

and line is the same as that in Fig. 3. In this case, the measured values in the temperature range of 100 to 400°C are observed to be fit within ± 10% scatter.

From the results discussed above, it can be confirmed that the differences of the measured values, which come from the differences between the results of the cyclic heat method and the transient hot-wire method, are within ± 10%.

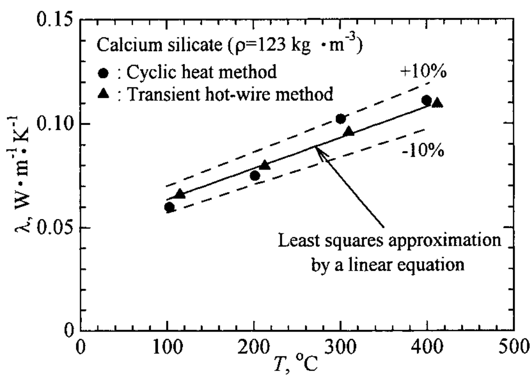

Fig. 4. Thermal conductivity λ of calcium silicate with a bulk density $\rho = 123$ kg·m^{-3}. ● measured by the cyclic heat method; ▲ measured by the transient hot-wire method; —— leasts square fit of the measured results; – – – ± 10% deviations from solid line.

4.2. Thermal Conductivities of Anisotropic Materials

Figures 5 and 6 show the measured thermal conductivities of the alumina silica blanket (bulk density, $\rho = 125\ \text{kg}\cdot\text{m}^{-3}$) and the rock wool (bulk density, $\rho = 121\ \text{kg}\cdot\text{m}^{-3}$), respectively. Here, the open-circle symbol represents the thermal conductivity in the direction of the thickness λ_y measured by the cyclic heat method, the open-triangle symbol represents the thermal conductivity λ_h measured by the transient hot-wire method, and the closed-square symbol represents the thermal conductivity in the direction of the plane (parallel to the oriented fibers) λ_x measured by the cyclic heat method. On the other hand, the solid, dashed-and-dotted, and two-dot chain lines show the fitted curves of λ_x, λ_y, and λ_h by the least-squares method, respectively. The broken and dotted lines represent the estimated results of λ_x obtained by substituting the fitted curve equations for λ_h and λ_y into Eq. (8) and Eq. (10), respectively.

As shown in Fig. 7, the specimen for λ_x measurement by the cyclic heat method was fabricated by cutting a blanket into several equal strips, rotating them by 90°, and reintegrating them into a rectangular block. No adhesives were applied to bond the strips together. Instead, the strips were held together by winding a steel wire of 0.3 mm diameter around them.

It is seen from Figs. 5 and 6 that the results estimated by Eq. (8) are consistent with the solid line within an error of $\pm 1.5\%$ for the alumina

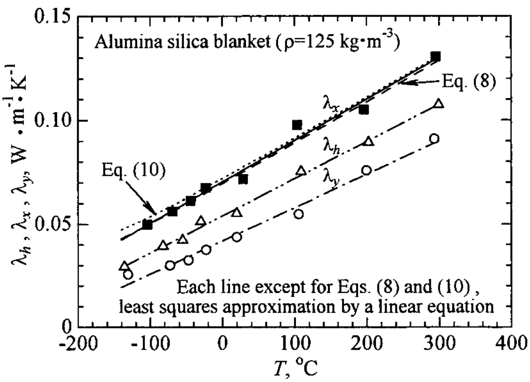

Fig. 5. Thermal conductivities λ_h, λ_x, and λ_y of alumina silica blanket with a bulk density $\rho = 125\ \text{kg}\cdot\text{m}^{-3}$. ○ λ_y measured by the cyclic heat method; △ λ_h measured by the transient hot-wire method; ■ λ_x measured by the cyclic heat method; —·—, —··— and —— least square fits, for ○, △ and ■; – – – estimated results by Eq. (8); ···· estimated results by Eq. (10).

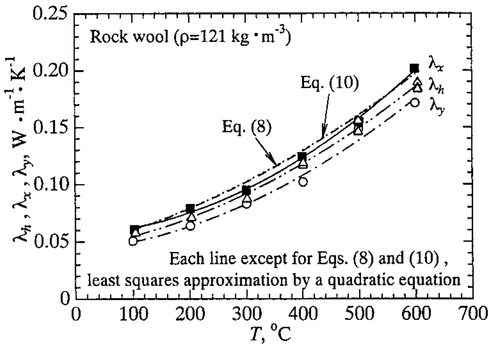

Fig. 6. Thermal conductivity λ_h, λ_x, and λ_y of rock wool with a bulk density $\rho = 121$ kg·m^{-3}. ○ λ_y measured by the cyclic heat method; △ λ_h measured by the transient hot-wire method; ■ λ_x measured by the cyclic heat method; –·–, –··– and —— least square fits for ○, △ and ■; – – – estimated results by Eq. (8); · · · · estimated results by Eq. (10).

silica blanket and within an error range of −3.2% (100°C) to 6.5% (290°C) for the rock wool. Therefore, the estimation method proposed here is considered to be adequate. Meanwhile, the estimated results using Eq. (10) agree with the solid line within an error range of 12% (−140°C) to 0.6% (300°C) for the alumina silica blanket and −2.7% (100°C) to 7.1% (285°C) for the rock wool. However, because of the elliptic integral included in Eq. (10), the calculations by Eq. (10) are more cumbersome in comparison with those represented by Eq. (8).

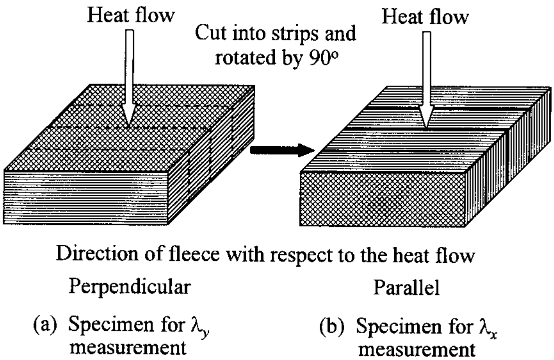

Fig. 7. Schematic of fibrous insulation.

5. CONCLUSIONS

It is shown that the thermal conductivity λ_h of anisotropic materials measured by the transient hot-wire method is expressed by $\lambda_h = \sqrt{\lambda_x \lambda_y}$, which is a geometrical mean of the thermal conductivities in the direction of the plane and the thickness λ_x, λ_y of the anisotropic materials. On the basis of the relational expression and measured results of λ_y and λ_h by the cyclic heat method and the transient hot-wire method, a simple estimation method for the plane directional thermal conductivity λ_x of fibrous heat insulations is proposed. From comparisons of the estimated results with the measured results on the alumina silica blanket and rock wool, the present estimation method is shown to be adequate. In this study, the cyclic heat method is applied for the λ_y measurements, but the plane directional thermal conductivity λ_x can also be estimated in the same manner by using other methods, for example, the guarded hot-plate method.

AKNOWLEDGMENTS

The authors wish to acknowledge the valuable suggestions offered by Professors I. Hatta of Nagoya University, E. Nemoto of Ibaraki National College of Technology, and B. B. Saha of Institute of Advanced Material Study, Kyushu University.

REFERENCES

1. T. Ohmura and M. Tsuboi, *Proc. 21st Japan Symp. Thermophys. Props.* (2000), p. 147.
2. J. Jackson, J. Adams, and R. C. Millar, *Thermal Transmission Measurements of Insulation, ASTM STP 660,* R. P. Tye, ed. (1977), p. 154.
3. S. E. Gustafsson, *Rev. Sci. Instrum.* **62**: 797 (1991).
4. T. Ohmura and M. Tsuboi, *Netsu Bussei* (*Japan J. Thermophysical Props.*) **13**:264 (1999).
5. T. Ohmura and M. Tsuboi, *Nichias Gijutsu Jihou* **317**:2 (2000).